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Global monodromy modulo 5 of quintic-mirror family

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Abstract

When we fix a symplectic basis of the third cohomology group of a nonsingular fibre of quintic-mirror family, the global monodromy Γ is a subgroup of symplectic group $\mathrm{Sp}(4,\mathbb{Z})$. The generators of Γ are well studied by Candelas, de la Ossa, Green and Parks. We consider calculating components of matrixes which are elements of Γ under this situation. In main result, elements of a subgroup Γ' of $\mathrm{GL}(4,\mathbb{Z})$ which is isomorphic to Γ are represented by components modulo 5.

Quintic-mirror family

Let $(x_1:\cdots:x_5)$ be the homogeneous coordinates of $\mathbb{P}^4$ and let $\psi\in\mathbb{P}^1=\mathbb{C}\cup\{\infty\}$. We give a hypersurface Q_ψ of $\mathbb{P}^4$ by

$$Q_\psi := \{x\in\mathbb{P}^4 \mid \textstyle\sum_{i=1}^5 {x_i}^5 - 5\psi\Pi_{i=1}^5 x_i = 0\}.$$

A finite group G acts on Q_ψ as follows.

$$\mu_5 : \text{the group of 5-th root of } 1\in\mathbb{C},$$

$$\widetilde{G} := (\mu_5)^5/\{(\alpha_1,\cdots,\alpha_5)\in(\mu_5)^5 \mid \alpha_1=\cdots=\alpha_5\},$$

$$G := \{(\alpha_1,\cdots,\alpha_5)\in\widetilde{G} \mid \alpha_1\cdots\alpha_5=1\},$$

$$G\times Q_\psi \to Q_\psi\ ;$$
$$((\alpha_1,\cdots,\alpha_5),(x_1:\cdots:x_5)) \mapsto (\alpha_1x_1:\cdots:\alpha_5x_5).$$

When we divide the hypersurface Q_ψ by G, canonical singularities appear. For $\psi\in\mathbb{C}\subset\mathbb{P}^1$, it is known that there is a simultaneous minimal desingularization of these singularities, and we have the family $(W_\psi)_{\psi\in\mathbb{P}^1}$ of the mirror to the above hypersurface. When ψ belongs to $\mu_5\subset\mathbb{C}\subset\mathbb{P}^1$, W_ψ has one ordinary double point. W_∞ is a normal crossing divisor in the total space. The other fibres of $(W_\psi)_\psi$ are smooth with Hodge numbers $h^{p,q}=1$ for $p+q=3,\ p,q\geq 0$.

By the action of

$$\alpha\in\mu_5,\ (x_1,\cdots,x_5)\ \mapsto (x_1,\cdots,x_4,\alpha^{-1}x_5),$$

we have the isomorphism from the fibre over ψ to the fibre over $\alpha\psi$. Let $\lambda=\psi^5$ and let

$$\begin{array}{ccc}(W_\lambda)_\lambda & = & ((W_\psi)_\psi)/\mu_5\\ \downarrow & & \downarrow\\ (\lambda\text{-plane}) & = & (\psi\text{-plane})/\mu_5.\end{array}$$

This family $(W_\lambda)_\lambda$ is so-called quintic-mirror family. Quintic-mirror family was constructed by Greene and Plesser.

Local monodromy

Let $b\in\mathbb{P}^1-\{0,1,\infty\}$. Candelas, de la Ossa, Green and Parks constructed a symplectic basis $\{A^1,A^2,B_1,B_2\}$ of $H_3(W_b,\mathbb{Z})$ and calculated the monodromies around $\lambda=0,1,\infty$ on the period integrals of a holomorphic 3-form on this basis. By the relation between the symplectic basis $\{\beta^1,\beta^2,\alpha_1,\alpha_2\}$ of $H^3(W_b,\mathbb{Z})$ which is the dual basis of $\{B_1,B_2,A^1,A^2\}$ and the period integrals, we have the matrix representations of the local monodromies for the basis $\{\beta^1,\beta^2,\alpha_1,\alpha_2\}$. We recall their results.

Matrix representations A,T,T_∞ of local monodromies around $\lambda=0,1,\infty$ for the basis $\{\beta^1,\beta^2,\alpha_1,\alpha_2\}$ are as follows:

$$A=\begin{pmatrix}11&8&-5&0\\5&-4&-3&1\\20&15&-9&0\\5&-5&-3&1\end{pmatrix},T=\begin{pmatrix}1&0&0&0\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix},$$

$$T_\infty=\begin{pmatrix}-9&-3&5&0\\0&1&0&0\\-20&-5&11&0\\-15&5&8&1\end{pmatrix}.$$

Global monodromy

Let $\langle\ ,\ \rangle$ be the anti-symmetric bilinear form on $H^3(W_b,\mathbb{Z})$ defined by the cup product. Global monodromy Γ is $\mathrm{Im}\Big(\pi_1(\mathbb{P}^1-\{0,1,\infty\})\to\mathrm{Aut}\big(H^3(W_b,\mathbb{Z}),\langle\ ,\ \rangle\big)\Big)$.

When we take $\{\beta^1,\beta^2,\alpha_1,\alpha_2\}$ as the basis $H^3(W_b,\mathbb{Z})$, $\mathrm{Aut}\big(H^3(W_b,\mathbb{Z}),\langle\ ,\ \rangle\big)$ is identified $\mathrm{Sp}(4,\mathbb{Z})$, and Γ is the subgroup of $\mathrm{Sp}(4,\mathbb{Z})$ which is generated by A and T.

Lemma There exisits $P\in\mathrm{GL}(4,\mathbb{Q})$ such that

$$P^{-1}A^{-1}P=\begin{pmatrix}1&1&0&0\\0&1&1&-1\\0&0&1&-1\\5&5&5&-4\end{pmatrix},\ P^{-1}T^{-1}P=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&1\\0&0&0&1\end{pmatrix}.$$

$\Gamma':=\{P^{-1}XP\in\mathrm{GL}(4,\mathbb{Z}) \mid X\in\Gamma\}$ is isomorphic to Γ as group.

Main result

Let $X\in\Gamma'$. Then, $X\equiv\begin{pmatrix}1&n&3n^2+2n&a\\0&1&n&b\\0&0&1&c\\0&0&0&1\end{pmatrix}\pmod 5.$

Here n,a,b,c is the elements of $\mathbb{Z}$.

References

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